VITA

HRUSHIKESH NARHAR MHASKAR

Education:

- Ph.D. in Mathematics; The Ohio State University, Columbus, Ohio, August 1980.
  - Advisor: Professor Geza Freud, (because of the sad demise of Professor Geza Freud on September 27, 1979, Professor Paul Nevai agreed to assume my formal advisorship).
- M.Sc. in Computer and Information Science, The Ohio State University, March 1980
- M.Sc. in Mathematics, The Ohio State University, August 1977.
- M.Sc. in Mathematics, The Indian Institute of Technology, Bombay, India, April 1976.

Teaching and Research Positions:

- Research professor, Claremont Graduate University, Claremont, 2012–Present
- Visiting Associate, California Institute of Technology, Pasadena, 2012–Present
- John von Neumann (distinguished) professor, Technical University, Munich, Spring, 2011.
- Professor Emeritus, The California State University, Los Angeles, 2012–Present.
- Professor; The California State University, Los Angeles, 1990-2012.
- Associate Professor; The California State University, Los Angeles, 1985-1990.
- Assistant Professor; The California State University, Los Angeles, 1980-1985.
- Visiting Professor; Katholische Universität, Eichstätt, Germany, Summer, 1990.
- Visiting Associate Professor, Texas A&M University, College Station, Fall, 1988.
- Associate Professor; Bowling Green State University, Bowling Green, Ohio, 1985-86.
- Visiting Assistant Professor; The University of Michigan, Ann Arbor, 1981-1982.
- Visiting Assistant Professor; The University of South Florida, Tampa, Fall, 1982.
- Visiting Scientist, The Indian Institute of Technology, Bombay, Fall 1983.

Honors and Awards:

1. Presidential Fellowship (Research Award), The Ohio State University, 1979-80.
2. Outstanding Young Men of America, 1985.
3. Meritorious Performance and Professional Promise Award, California State University, Los Angeles, 1988, 1989.
4. Humboldt Research Fellowhip, 1992, 1993, 1994, 1998, 2006.
5. Who's Who in America's Teachers, 1994.
6. Summer Faculty Research Program, Hanscomb Air Force Base, Boston, Summer, 1996.
7. John von Neumann Professor, Technical University of Munich, Spring, 2011.

Grants:

1. California State University Foundation, Institutional Grant, Fall 1984.
2. Affirmative Action Faculty Development Grant, California State University, 1987.
3. National Security Agency Contract, 1991.
4. National Science Foundation 1994-97.
5. Research and Development Laboratories, 1997.
6. Air Force Office of Scientific Research : 1993-96, 1997-99.
7. National Science Foundation, 1999-2002, 2002-2005, 2006–2009, 2009–2012.
8. U. S. Army Research Office, 2001-2004, 2004-2009, 2009–2012, 2015–2018.

Professional Activities:

- Member of Editorial Board : (1) Journal of Approximation Theory (2) Jaen Journal of Approximation (3) Mathematics of Computation and Data Science
- NSF Review Panel June 2008
- Google scholar H-index: 31, 04/01/2016, 3463 citations.

Thesis supervision

- Dominik Schmid, Technical University of Munich, Ph. d., 2009 (co-adviser with Dr. Frank Filbir)
- Eugene Shvarts, California State University, Los Angeles, M. S., 2012.

Highlights of research

Potential theory and orthogonal polynomials: Together with E. B. Saff, we proved that the supremum norm on the whole real line of an expression of the form $\exp(-Q(x))P(x)$, where P is a polynomial of degree n is attained on an interval of the form $[-a_n, a_n]$, *independently of* P, giving precise expressions for a_n, often known in the literature as **Mhaskar–Rakhmanov–Saff number**. The techniques which were introduced here blossomed into a full theory, described in a treatise by Saff and Totik. Major applications include the study of asymptotic behavior of the zeros and leading coefficients of a large class of orthogonal polynomials on the whole real line. We proved some long standing conjectures by Erdős and Freud on this subject.

Neural and RBF networks: At the start of our research, there was no theory for the degree of approximation by neural networks for functions in Sobolev classes, in spite of the wide recognition of the fact that their universal approximation property is what makes them most useful. The theory of RBF approximation was dominated by themes from spline approximation, so that the degree of approximation was estimated in terms of a scale parameter. Together with different coauthors, we introduced a unified theory of both neural and RBF networks, and established a deep connection of this theory with that of polynomial approximation. In particular, we gave an explicit, universal construction for the networks to **guarantee** the optimal rate of approximation of smooth functions. Our constructions do not require optimization of any kind, and are therefore free of the many problems associated with this classical approach. Together with different coauthors, we extended this theory for Gaussian networks for approximation on the whole Euclidean space, zonal function networks for approximation on the sphere, and "eignets" for approximation on arbitrary, smooth, compact manifolds. In particular, there is a long standing puzzle as to why the theoretical bounds are much worse than what the engineers can accomplish in practice. We established **converse theorems** proving that if any method of approximation by such networks yields a certain rate of approximation, necessarily, the target function must be smooth and our constructions would give the same rate of approximation for the individual target function involved.

Polynomial frames: It is well known that an algebraic (or trigonometric) polynomial is completely determined by its values on a small interval. Therefore, such traditional methods of function approximation as least squares fit, Fourier projection, and interpolation are unsatisfactory for approximating functions which are generally "good" but have a few "bad points". Together with C. K. Chui and J. Prestin, we laid the foundations of a theory of localized polynomial frames. We obtained wavelet–like expansions of functions on different domains utilizing such global information as Fourier coefficients or samples at randomly selected points throughout the domain of the target function. The local behavior of the terms in these expansions in the neighborhood of a point in the domain **characterize** the local smoothness class to which the function belongs in the neighborhood. The constructions have been generalized to the context of Jacobi expansions, spherical polynomial expansions, and expansions in terms of eigenfunctions of a suitable partial differential operator on a manifold. Applications include multi–source direction finding in phased array antennas, sparse representation of functions, representation of functions using finitely many bits, solutions of pseudo–differential equations on the sphere, velocity estimation in the gulf stream, terrain data modeling. Current work in this direction is focused on finding faster and more efficient variants of the finite difference schemes for the solution of partial differential equations in various settings.

Modeling on data dependent manifolds: Many recent applications require the analysis of high dimensional, unstructured data sets. While the previous works on the subject focused on the understanding of the geometry of the data set, our techniques enable us to obtain answers to the queries which can often be formulated as modeling of functions on an **unknown** metric measure space. Motivated by the work of Coifman and Lafon, and Belkin and Niyogi, we have developed novel techniques to extend our previous work on known manifolds to this setting. In particular, we have obtained wavelet–like expansions of the target functions using RBF–like structures called "eignets" and iterates of a heat kernel or Green kernel. Applications include pattern recognition, in particular, recognition of hand–written digits and context sensitive image completion. Potential impact areas are homeland security, financial applications, biological and genetic research, image processing, pattern classification and chemometry. Future work in this direction includes a similar analysis on graphs and smooth extensions of functions on finite sets.

List of Publications
Hrushikesh Narhar Mhaskar

Books:

1. Weighted Polynomial Approximation, World Scientific, Singapore, 1996.
2. Guest Editor : Mathematical Aspects of Neural Networks, Special Issue of Advances in Computational Mathematics, **5** (1996) (With C. A. Micchelli).
3. Fundamentals of Approximation Theory, Narosa Publishing Co., Delhi, 2000 (With D. V. Pai).
4. Local analysis of spectral and scattered data; in preparation, to be publised by Atlantis Publications (Springer Verlag).
5. Representation of Functions on Big Data, in preparation, Atlantis Press (Springer Verlag) (With C. K. Chui).
6. Co-Editor: Wavelet Analysis and Applications, Proceedings of the international workshop in Delhi, 1999, Narosa Publishing Co., Delhi, 2001. (With P. K. Jain, M. Krishnan, J. Prestin, and D. Singh).
7. Co-Editor: Frontiers in interpolation and approximation, Dedicated to the memory of Ambikeshwar Sharma, Chapman and Hall/CRC, Boca Raton, Florida, 2006 (With N. K. Govil, R. N. Mohapatra, Z. Nashed, and J. Szabados).
8. Co-Editor: Special volumes of **Journal of Approximation Theory** in memory of George G. Lorentz, Volumes 158, 159 (two issues each), Elsevier, 2009 (With C. K. Chui)

Articles:

1. Weighted polynomial approximation in rearrangement invariant Banach function spaces on the whole real line; Indian J. Math., **22** (3)(1980), 209-224 (with G. Freud).
2. K-functionals and moduli of continuity in weighted polynomial approximation; Arkiv for Mat., **21** (1983), 145-161 (with G. Freud).
3. Weighted analogues of Nikolskii-type inequalities and their applications; in Proc. Conference on Harmonic Analysis in Honor of A. Zygmund (Becker, et al., eds.), Vol. II (1983), Wadsworth International: Belmont, 783-801.
4. Weighted polynomial approximation of entire functions, I; J. of Approximation Theo., **35** (1982), 203-213.
5. Weighted polynomial approximation of entire functions, II; J. of Approximation Theo., **33** (1981), 59-68.
6. Comonotone approximation by splines of piecewise monotone functions: J. of Approximation Theo. **35** (1982) 364-369 (with D. Leviatan).
7. The rate of monotone spline approximation in the L_p-norm; SIAM J. of Math. Anal., **13**(5) 1982, 866-874 (with D. Leviatan).
8. On a problem of G. Freud; J. of Math, Anal. and Appl., **96** (1983), 395-404.
9. On the domain of convergence of expansions in polynomials orthogonal with respect to general weight functions on the whole real line; Acta. Math. Acad. Sci. Hungar., **44**(3-4) (1984), 223-227.
10. Extremal problems associated with polynomials with exponential weights; Trans. Amer. Math. Soc., **285** (1984), 223-234 (with E.B. Saff).
11. Extremal problems for polynomials with Laguerre weights; in Approximation Theory, IV; College Station Texas, 1983, (Chui, et. al. eds.), Academic Press, 1983, 619-624 (with E.B. Saff).

12. Polynomials with Laguerre weights in L_p; in Proc. Conf. Rational approximation and interpolation, Tampa, Florida, 1983, (Graves-Morris, et. al. eds.) 511-523 (with E.B. Saff).

13. On the smoothness of Fourier transform; in the Proc. International Symposium on Interpolation spaces, Lund. Sweden, 1983 (Cwickel, Peetre, eds.), Lecture Notes, 1070, Springer Verlag, Berlin, 1984, 202-207.

14. A trace theorem for caloric functions; International Journal of Math and Mathematical Sciences, B(1) (1985), 29-35.

15. Extensions of Dirichlet-Jordan convergence criterion to a general class of orthogonal polynomial expansions; J. of Approx. Theo., **42** (1984), 138-148.

16. Where does the sup norm of a weighted polynomial live? (A Generalization of incomplete polynomials); Constructive Approximation, **1** (1985), 71-91 (with E.B. Saff).

17. Weighted polynomials on finite and infinite intervals; A unified approach; Bulletin American Mathematical Society **11** (1984), 351-354 (with E.B. Saff).

18. Weighted polynomial approximation; J. of Approx. Theo., **46** (1986), 100-110 (invited survey paper).

19. A Weierstrass-type theorem for certain weighted polynomials; in Approximation Theory and Applications (S.P. Singh ed.), Pitman, Boston, 1985, 115-123 (with E.B. Saff).

20. A quantitative Dirichlet-Jordan type theorem for orthogonal polynomial expansions; SIAM J. of Math., Anal., **19** (1988), 484-492.

21. Where does the L_p-norm of weighted polynomial live?, Trans. Amer. Math. Soc., **303** (1987), 109-124, Errata: **308** (1988), 431. (with E.B. Saff).

22. Freud's conjecture for exponential weights, Bull. Amer. Math. Soc., **15** (1986), 217-221 (with D.S. Lubinsky and E.B. Saff).

23. A proof of Freud's conjecture for exponential weights, Const. Approx. **4** (1988), 65-83. (With D.S. Lubinsky and E.B. Saff).

24. A weighted transfinite diameter, in Approximation Theory V (Chui et. al. eds.), Academic Press, 1986, 479-481.

25. A rate of convergence theorem for expansions in Freud polynomials, J. Approx. Theo. **55** (1988), 150-171.

26. Some Discrepancy theorems; in "Approximation Theory, Tampa" (E.B. Saff ed.) Vol. 1287, Lecture Notes in Mathematics, Springer Verlag, Berlin, 1987, 117-131.

27. Weighted analogue of capacity, transfinite diameter and Chebyshev constant;
Constr. Approx., **8** (1991), 105-124. (with E.B. Saff)

28. The distribution of zeros of asymptotically extremal polynomials; J. Approx. Theo. **65** (1991), 279–300 (with E.B.Saff).

29. The convergence of Fourier series and a K-functional; J. Math. Anal. Appl., **154** (1991), 134-141.

30. Approximation in certain intermediate spaces; J. Approx. Theory, **62** (1990), 110-132.

31. On the distribution of zeros of polynomials orthogonal on the unit circle; J. Approx. Theory, **63** (1990), 30-38. (with E.B. Saff).

32. Bounds for certain Freud polynomials; J. Approx. Theory, **63** (1990), 238-254.

33. On the n-width for weighted approximation of entire functions; in Approximation Theory VI, (Chui et al eds.), Academic Press, 1989, 429-432. (With C.A. Micchelli.)

34. Hermite interpolation at the zeros of Freud polynomials; Acta Math. Hung., **60** (1992), 225-240 (With Y. Xu).

35. The rate of convergence of a Hermite interpolation process; in Approximation Theory VI, (Chui et al eds.), Academic Press, 1989, 433-436. (With Y. Xu)

36. A general study of maximal robust stability regions; Circuits, Systems, and Signal Processing, **10** (1991), 15-30 (With C.K. Chui).

37. The mean convergence of expansions in Freud-type orthogonal polynomials; SIAM J. Math. Anal., **22** (1991), 847-855 (With Y. Xu).

38. On multivariate robust stability; SIAM J. Control and Optimization, **30** (1992), 1190-1199 (With C.K. Chui).

39. General Markov-Bernstein and Nikolskii-type inequalities; Approximation Theory and its Applications, **6:4** (1990), 107–117.

40. Finite-infinite range inequalities in the complex plane; Int. J. Math. and Math. Sci., **14** (1991), 625-638.

41. Weighted polynomials, radial basis functions and potentials in locally compact spaces; Num. Funct. Anal. and Optimization, **11** (9& 10), 1990-91, 987–1017.

42. A general discrepancy theorem; Arkiv för Mathematik, **31** (1993), 219-246 (With H.–P. Blatt).

43. Detection of singularities using segment approximation; Mathematics of Computation, **59** (1992), 533-540 (With R. Grothmann).

44. On trigonometric wavelets; Constr. Approx., **9** (1993), 167-190 (With C.K. Chui).

45. Approximation by superposition of a sigmoidal function and radial basis functions; Advances in Applied Mathematics, **13** (1992), 350-373 (With C.A. Micchelli).

46. Weighted approximation of entire functions on unbounded subsets of the complex domain; Bull. Canadian Math. Soc., **36** (1993), 303-313.

47. Approximation properties of a multilayerd feedforward artificial neural network; Advances in Computational Mathematics, **1** (1993), 61-80.

48. Neural networks for localized approximation; Mathematics of Computation, **63** (1994), 607-623 (With C. K. Chui and X. Li).

49. Neural networks for localized approximation of real functions; In "Neural Networks for Signal Processing III", (C.A. Kamm et. al. eds.), IEEE, New York, 1993, 190-196.

50. Dimension-independent bounds on approximation by neural networks; IBM J. of Research and Development,**38** (1994), 277-284 (With C. A. Micchelli).

51. How to choose an activation function; in "Neural Information Processing Systems, 6", (J. D. Cowan, G. Tesauro, J. Alspector Eds.), Morgan Kaufmann Publishers, San Fransisco, 1993, pp. 319-326 (With C. A. Micchelli).

52. A discrepancy theorem concerning polynomials of best approximation in $L^p_w[-1,1]$; Monatshafte Mathematik, **120** (1995), 91-103. (With H.–P. Blatt).

53. Approximation of real functions using neural networks; in "Proc. of Int. Conf. on Computational Mathematics, New Delhi, India, 1993", (C. A. Micchelli ed.), World Scientific, 1994, pp. 267-278.

54. Degree of approximation by neural and translation networks with a single hidden layer, Advances in Applied Mathematics, **16** (1995), 151-183. (With C. A. Micchelli).

55. Neural networks for optimal approximation of smooth and analytic functions; Neural Computation, **8** (1996), 164- 177.

56. Versatile Gaussian networks; Proceedings of IEEE Workshop on Nonlinear Image and Signal Processing, (I. Pitas Editor), Halkidiki, Greece, June, 1995, IEEE, pp.70-73.

57. Some discrepancy theorems in the theory of weighted polynomial approximation; Journal of Mathematical Analysis and Applications, **219** (1998), 312–330 (With H.–P. Blatt).

58. Limitations of the approximation capabilities of a neural network with a single hidden layer, Advances in Computational Mathematics, **5** (1996), 233-243. (With C. K. Chui and X. Li).

59. On smooth activation functions; in "Mathematics of Neural Networks, Models, Algorithms, and Applications", (S. W. Ellacott, J. C. Mason, and I. J. Anderson Eds.), Kluwer Academic Publishers, 1997, pp. 275–279.

60. Neural networks for function approximation; in "Neural networks for signal processing, V", (F. Girosi, J. Makhoul, E. Manolakos, E. Wilson Eds.), IEEE, New York, 1995, pp.21-29. (With L. Khachikyan).

61. Neural networks for functional approximation and system identification; Neural Computation, **9** (1997), 143–159. (With N. Hahm)

62. System identification using neural networks; in "Neural networks for signal processing, VI", (S. Usui, Y. Tohkura, S. Katagiri, and E. Wilson Eds.), pp. 82–88, IEEE, New York, 1996, (With N. Hahm).

63. Bounded quasi-interpolatory polynomial operators; Journal of Approximation Theory, **96** (1999), 67–85. (With J. Prestin).

64. On Marcinkiewicz-Zygmund-Type Inequalities; in "Approximation theory: in memory of A. K. Varma", (N. K. Govil, R. N. Mohapatra, Z. Nashed, A. Sharma, and J. Szabados Eds.), Marcel Dekker, 1998, pp.389–404. (With J. Prestin)

65. On a choice of sampling nodes for optimal approximation of smooth functions by generalized translation networks; in Artificial Neural Networks, Conference Publication No. 440 (IEE), 1997, pp. 210-215. (With J. Prestin)

66. Neural beam-steering and direction finding; in Neural Networks in Engineering Systems, (A. B. Bulsari and S. Kallio Eds.), Royal Institute of Technology, Stockholm, 1997, pp. 269-272. (With H. Southall)

67. Approximation of smooth functions by neural networks; *Invited survey,* in "Dealing with complexity: A neural network approach", (K. Warwick et. al. eds), "Perspectives in Neural Computing", Springer Verlag, London, 1998, pp.189–204.

68. Polynomial frames for the detection of singularities; in "Wavelet Analysis and Multiresolution Methods" (Ed. Tian-Xiao He), Lecture Notes in Pure and Applied Mathematics, Vol. 212, Marcel Decker, 2000, 273–298. (With J. Prestin).

69. On a sequence of fast decreasing polynomial operators; in: Applications and Computation of Orthogonal Polynomials (Eds. W. Gautschi, G.H. Golub, G. Opfer) Internat. Ser. Numer. Math., Birkhäuser, Basel, 1999, 165-178. (With J. Prestin).

70. On the detection of singularities of a periodic function; Advances in Computational Mathematics, **12** (2000), 95–131 (With J. Prestin).

71. Spherical Marcinkiewicz-Zygmund inequalities and positive quadrature. Math. Comp. **70** (2001), no. 235, 1113–1130. (With F. J. Narcowich and J. D. Ward). (Corrigendum: Math. Comp. **71** (2001), 453–454.)

72. Approximation Properties of Zonal Function Networks Using Scattered Data on the Sphere; Advances in Computational Mathematics, **11** (1999), 121–137 (With F. J. Narcowich and J. D. Ward).

73. Representing and analyzing scattered data on the sphere; in "Multivariate approximation and applications" (A. Pinkus, D. Leviatan, N. Dyn, and D. Levin Eds.), Cambridge University Press, Cambridge, 2001, pp. 44–72. (With F. J. Narcowich and J. D. Ward).

74. On a build-up polynomial frame for the detection of singularities; in "Self-Similar Systems" (V. B. Priezzhev and V. P. Spiridonov Eds.), Joint Institute for Nuclear Research, Dubna, Russia, 1999, pp. 98–109. (With J. Prestin).

75. Quasi-interpolation in shift invariant spaces; J. Math. Anal. and Appl. **251** (2000), 356-363. (With F. J. Narcowich and J. D. Ward).

76. Polynomial frames on the sphere; Adv. Comput. Math. 13 (2000), no. 4, 387–403. (With F. J. Narcowich, J. Prestin, and J. D. Ward).

77. Zonal function network frames on the sphere; Neural Networks, **16**(2) (2003), 183–203 (With F. J. Narcowich and J. D. Ward).

78. Approximation theory and neural networks; in "Wavelet Analysis and Applications, Proceedings of the international workshop in Delhi, 1999" (P. K. Jain, M. Krishnan, H. N. Mhaskar J. Prestin, and D. Singh Eds.), Narosa Publishing, New Delhi, India, 2001, 247–289.

79. Neural network frames on the sphere; in "Neural networks for signal processing, X, (B. Widrow, L. Guan, K. Paliwa, T. Adall, J. Larsen, E. Wilson, and S. Douglas Eds.), IEEE, New York, 2000, pp. 175–184. (With F. J. Narcowich and J. D. Ward).

80. Approximation with Interpolatory Constraints; Proc. Amer. Math. Soc. **130** (2002), no. 5, 1355–1364 (With F. J. Narcowich, N. Sivakumar, and J. D. Ward).

81. A local discrepancy theorem; Indiagationes Math., **12**(1), (2001), 23–39. (With V. V. Andrievskii and H.-P. Blatt).

82. On the representation of band limited functions using finitely many bits; Journal of Complexity, **18** (2002), no. 2, 449–478.

83. On the degree of approximation in multivariate weighted approximation; in "Advanced Problems in Constructive Approximation" (Proceedings of the IDOMAT 2001 conference) (M.D. Buhmann and D.H. Mache Eds.), ISNM **142**, Birkhäuser, Basel, 2002, pp.129–141.

84. On the representation of band-dominant functions on the sphere using finitely many bits; Advances in Computational Mathematics, **21** (2004), 127–146. (With F. J. Narcowich and J. D. Ward).

85. When is approximation by Gaussian networks necessarily a linear process?; Neural Networks, **17** (2004), 989–1001.

86. Local quadrature formulas on the sphere; Journal of Complexity, **20** (2004), 753–772 (**Among the top 25 most downloaded articles, October–December 2004)**).

87. A tribute to Géza Freud; Journal of Approximation Theory, **126** (2004), 1–15. (Invited paper).

88. On the tractability of multivariate integration and approximation by neural networks; Journal of Complexity, **20** (2004), 561–590 (**Among the top 25 most downloaded articles, October–December 2004)**).

89. Local quadrature formulas on the sphere, II; in "Advances in Constructive Approximation" (M. Neamtu and E. B. Saff eds), Nashboro Press, Nashville, 2004, pp. 333–344.

90. On local smoothness classes of periodic functions; Journal of Fourier Analysis and Applications, **11** (3) (2005), 353 - 373 (with J. Prestin).

91. Polynomial operators and local smoothness classes on the unit interval; Journal of Approximation Theory, **131**(2004), 243-267.

92. On the representation of smooth functions on the sphere using finitely many bits; Applied and Computational Harmonic Analysis **18**, Issue 3 , May 2005, Pages 215-233 (**Among the top 25 most downloaded articles, April-September 2005**)..

93. A Markov–Bernstein inequality for Gaussian networks; in "Trends and applications in constructive approximation" (M. G. de Bruin, D. H. Mache, and J. Szabados eds.), ISNM **105**, Birkhäuser Verlag, Bassel, 2005, pp. 165–180.

94. Polynomial frames: a fast tour; in "Approximation Theory XI, Gatlinburg, 2004" (C. K. Chui, M. Neamtu, and L. Schumaker Eds.), Nashboro Press, Brentwood, 2005, 287–318. (With J. Prestin) (Invited paper).

95. Polynomial operators and local approximation of solutions of pseudo-differential equations on the sphere; Numerische Mathematik, **103** (2006), 299–322. (With Q. T. Le Gia).

96. Matrix–free interpolation on the sphere; SIAM J. Numer. Analysis **44** (3) (2006), pp. 1314–1331. (With M. Ganesh).

97. Weighted quadrature formulas and approximation by zonal function networks on the sphere; Journal of Complexity Theory, **22** (3), June 2006, 348–370 (**Among the top 25 most downloaded articles, July–September 2005**).

98. Quadrature–free quasi–interpolation on the sphere; Electronic Transactions on Numerical Analysis,**25** (2006), 101–114 (With M. Ganesh).

99. On bounded interpolatory and quasi-interpolatory polynomial operators; in "Frontiers in interpolation and approximation" (N. K. Govil, H. N. Mhaskar, R. N. Mohapatra, Z. Nashed, and J. Szabados eds.), Chapman and Hall/CRC, Boca Raton, 2006, pp. 345–364 (Invited paper).

100. Quadrature in Besov spaces on the Euclidean sphere; Journal of Complexity, **23** (2007), 528–552 (With K. Hesse and I. H. Sloan) (**Among the top 25 most downloaded articles, October–December 2007)**).

101. Polynomial operators for spectral approximation of piecewise analytic functions; Appl. Comput. Harmon. Anal. **26** (2009) 121–142 (With J. Prestin).

102. Diffusion polynomial frames on metric measure spaces; Applied and Computational Harmonic Analysis, Volume 24, Issue 3, May 2008, Pages 329-353 (With M. Maggioni) (**Among the top 25 most downloaded articles, April-June 2008**).

103. Localized linear polynomial operators and quadrature formulas on the sphere; SIAM J. Numer. Anal. **47** (1) (2008), 440–466. (With Q. T. Le Gia).

104. Polynomial operators and local smoothness classes on the unit interval, II; Jaén J. of Approx., **1** (1) (2009), 1–25. (Invited paper)

105. On a filter for exponentially localized kernels based on Jacobi polynomials; Journal of Approximation Theory 160 (2009), pp. 256-280 (With F. Filbir and J. Prestin)

106. A quadrature formula for diffusion polynomials corresponding to a generalized heat kernel; Journal of Fourier Analysis and Applications 16 (2010), 629–657 (with F. Filbir).

107. L^p Bernstein estimates and approximation by spherical basis functions; Math. Comp. 79 (2010), no. 271, 1647–1679. (With F. Narcowich, J. Prestin, and J. D. Ward)

108. Eignets for function approximation; Appl. Comput. Harmon. Anal. 29 (2010) 63–87.

109. MRA Contextual-Recovery Extension of Smooth Functions on Manifolds; Appl. Comput. Harmon. Anal. 28 (2010) 104–113 (With C. K. Chui).

110. Minimum Sobolev norm interpolation with trigonometric polynomials on the torus; Journal of Computational Physics, **249** (2013) 96–112 (With S. Chandrasekaran and K. R. Jayaraman).

111. Minimum Sobolev Norm schemes and applications in image processing. In IS&T/SPIE Electronic Imaging, International Society for Optics and Photonics, (2010, February), pp. 753507-753507 (with Chandrasekaran, S., Jayaraman, K. R., Moffitt, J., and Pauli, S.).

112. Higher order numerical discretization methods with Sobolev norm minimization, Accepted for publication in the Proceedings of International Conference on Computational Science, ICCS 2011, Procedia Computer Science, Elsevier (With S. Chandrasekaran, K. R. Jayaraman, M. Gu, and J. Moffitt).

113. A generalized diffusion frame for parsimonious representation of functions on data defined manifolds; Neural Networks **24** (2011) 345–359.

114. Wiener type theorems for Jacobi series with nonnegative coefficients; Proceedings of the American Mathematical Society, **140**(3) (2012), 977–986 (With S. Tikhonov).

115. Marcinkiewicz–Zygmund measures on manifolds; Journal of Complexity, **27** (2011), 568–596 (With F. Filbir).

116. On the problem of parameter estimation in exponential sums; Constructive Approximation **35** (3)(2012), 323–343 (With F. Filbir and J. Prestin).

117. Learning Biomedical Data Locally using Diffusion Geometry Techniques; in Proceeding (771) Imaging and Signal Processing in Health Care and Technology / 772: Human–Computer Interaction / 773: Communication, Internet and Information Technology – 2012, (R. Merrell, D.-G. Shin, M.H. Hamza Eds) Baltimore, 2012, pp. 125–131. (With M. Ehler, F. Filbir)

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119. Locally learning biomedical data using diffusion frames; Journal of Computational Biology, **19**, (11) (2012), 1251–1264 DOI: 10.1089/cmb.2012.0187 (With M. Ehler, F. Filbir).

120. Smooth function extension based on high dimensional unstructured data; Mathematics of Computation, **290**(83)(2014), 2865–2891 (With C. K. Chui).

121. Applications of classical approximation theory to RBF networks and computational harmonic analysis; Bull. Math. Sci. 3 (2013), 485–549 (With P. Nevai and E. Shvarts).

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123. Representation of functions on big data: graphs and trees; Applied and Computational Harmonic Analysis, Volume 38, Issue 3, May 2015, Pages 489-509 (With C. K. Chui and F. Filbir).

124. Local numerical integration on the sphere; Int. J. Geomath 5 (2014),143–162 (with J. Beckman and J. Prestin).

125. A minimum Sobolev norm technique for the numerical discretization of PDEs; Journal of Computational Physics 299 (2015) 649–666 (with S. Chandrasekaran).

126. Signal decomposition and analysis via extraction of frequencies; Appl. Comput. Harmon. Anal., **40** (2016) 97–136. (with C. K. Chui).

127. Localized summability kernels for Jacobi expansions; Accepted for publication in "Mathematical analysis, approximation theory, and their applications" (T. M. Rassias and V. Gupta Eds.), Springer Verlag.

128. Deep nets for local manifold learning; Submitted for publication (with C. K. Chui).

129. Learning Real and Boolean Functions: When Is Deep Better Than Shallow; Submitted for publication (with Q. Liao and T. Poggio).

130. Local approximation using Hermite functions; Submitted for publication.

Major invited lectures

1. International conference on computational mathematics, Delhi, India, December, 1992.
2. International conference on algorithmic aspects of wavelets and approximation methods, Gross Plasten, Germany, March, 1996.
3. International Symposium on Self-similar systems, Dubna, Russia, August, 1998.
4. International Workshop on Orthogonal Polynomials, Ballenstaedt, Germany, April, 1999.
5. International Conference on Approximation Theory, Kiev, Ukraine, May, 1999.
6. International Workshop on Wavelet Analysis and related topics, Delhi, India, August, 1999.
7. Ambikeshwar Sharma Lecture, University of Lucknow, Lucknow, India, August, 1999.
8. International conference on Approximation Theory, Bommerholtz, Germany, August, 2001.
9. Conference on Transform Techniques, Irinjalkuda, India, December, 2001.
10. Ramanujan Lecture, Indian Institute of Technology, Chennai, India, December, 2001.
11. International conference on complex systems, Nashua, New Hampshire, June, 2002.
12. Erdős Professor, Freud memorial lecture, Hungarian Academy of Sciences, Budapest, Hungary, September, 2002.
13. International symposium on wavelet analysis, St. Petersberg, Russia, July, 2003.
14. International workshop on approximation on the sphere, Nashville, Tennessee, December, 2003.
15. International Dortmund/Bochum meeting on approximation theory, Bommerholtz, Germany, February, 2004.
16. Eleventh international symposium on approximation theory, Gatlinburg, Tennessee, May, 2004.
17. Saff Festschrift, Atlanta, Georgia, November, 2004.
18. Conference on applications of approximation theory, Perth, Australia, April, 2005.
19. International conference on approximation theory, Ubeda, Spain, June, 2005.
20. International conference on multivariate approximation, Bommerholtz, Germany, September, 2008
21. International workshop honoring R. S. Varga, Kent, Ohio, 2008
22. International workshop on approximation theory and signal analysis honoring P. L. Butzer, Lindau, Germany, 2009.
23. International conference on approximation theory, Ubeda, Spain, 2009.
24. International conference on approximation theory, Ubeda, Spain, 2011.
25. International conference on approximation theory and harmonic analysis, Barcelona, Spain, 2011.
26. International conference on applied mathematics, May 2016.

Tentative plan of lectures
H. N. Mhaskar

I am thinking of giving 4 lectures, 100 min each (2 lectures per week at flexible times). The lectures would be in two parts: overview for 50 mins with slides, sketch of details on a document projector for 50 mins.

Topics:

1. Classical trigonometric approximation, localized kernels and applications : problem of parameter estimation, separation of blind source signals [17, 18, 5, 2].
2. Analogues for orthogonal polynomials and applications : Approximation on the sphere, drussen classification in Age–related Macular Degeneration, prediction of blood sugar from continous glucose monitoring readings [10, 14, 6, 4, 15].
3. Neural and RBF networks [9, 16, 12, 13, 11]
4. Approximation on data defined manifolds and graphs [7, 4, 8, 1, 3].

The citations are meant to give examples of some of the topics I will talk about. I will, of course, feel free to add or omit material.

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